Moment Equations for the Boltzmann Gas of Elastic Spheres: A Road from Statistical Physics to Continuum Mechanics

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Abstract. We describe a hierarchy of formal representations of the Fourier transform **F[f]** of a solution **f** of the Boltzmann equation for a gas of elastic spheres .The approximations are based on an original solution of the Hamburger moment problem where one attempts to recover **f** knowing the finite sequences of its first moments that describe the macroscopic properties of gas. In particular, we describe a hierarchy of the weighted power series representations for **F[f]** with coefficients that depend on the moments of **f** alone. The constructed expansions, contrary to ordinary power series expansions, can be Fourier inverted term by term, to recover the series representation of **f**. The solution of the Hamburger moment problem culminates in a quantitive, precise description of the Chapman- Enskog hypothesis for the density **f**.

The first two representations correspond to the Maxwellian and Gaussian expansions. They have been exploited by Grad, Levermore in their study of the Boltzmann equation. The next representation has a weight that depends on the first 13 moments of the Boltzmann density **f** and it yields modified Grad's 13 moment equations of gas dynamics. The principal tools in deriving the moment equations are the exact form of the Fourier transform of the nonlinear Boltzmann equation -analogous, in spirit, to Bobylev work on the Fourier transform of the Boltzmann equation for the Maxwell molecules- and the precise truncation criterion that is based on the reminder term for the finite Hamburger expansion. We also show that the solution of the Hamburger problem yields the boundary conditions for the moments that are computed from the microscopic, Kuscer type boundary conditions for the Boltzmann equation itself.

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